Fourth ABC Index and Fifth GA Index of Certain Special Molecular Graphs

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***Abstract*— Several chemical indices have been introduced in theoretical chemistry to measure the properties of molecular structures, such as atom bond connectivity index and geometric-arithmetic index. In this paper, we present the fourth atom bond connectivity index and fifth geometric-arithmetic index of fan molecular graph, wheel molecular graph, gear fan molecular graph, gear wheel molecular graph, and their *r*-corona molecular graphs.**

***Index Terms*— fourth atom bond connectivity index, fifth geometric-arithmetic index, *r*-corona molecular graph**

I. INTRODUCTION

Atom bond connectivity index, geometric-arithmetic index and other chemical indices are introduced to reflect certain structural features of organic molecules (See Yan et al., [1-2], Gao et al., [3-4], Gao and Shi [5], Gao and Wang [6], Xi and Gao [7-8], Xi et al., [9], Gao et al., [10] for more detail). The notation and terminology used but undefined in this paper can be found in [11].

M. Ghorbani et al., introduced the fourth Atom-Bond Connectivity index $ABC_4(G)$ [12] as

$$ABC_4(G)=\sum_{uv\in E(G)}\sqrt{\frac{S(u)+S(v)-2}{S(u)S(v)}},$$

where $S(v)=\sum_{uv\in E(G)} d(u)$. More results on fourth ABC index can refer to [13-15].

Similarity, the fifth geometric-arithmetic index was denoted by [16] as

$$GA_5(G)=\sum_{uv\in E(G)}\frac{2\sqrt{S(u)S(v)}}{S(u)+S(v)}.$$

More conclusions on GA_5 can refer to [17-18].

We define the general fifth geometric-arithmetic index as $GA_5^k(G)=\sum_{uv\in E(G)}\left(\frac{2\sqrt{S(u)S(v)}}{S(u)+S(v)}\right)^k$.

Let P_n and C_n be path and cycle with n vertices. The molecular graph $F_n=\{v\}\vee P_n$ is called a fan molecular graph and the molecular graph $W_n=\{v\}\vee C_n$ is called a wheel molecular graph. Molecular graph $I_r(G)$ is called r- crown molecular graph of G which splicing r hang edges for every vertex in G. By adding one vertex in every two adjacent vertices of the fan path P_n of fan molecular graph F_n, the resulting molecular graph is a subdivision molecular graph called gear fan molecular graph, denote as $\tilde{F}_n$. By adding one vertex in every two adjacent vertices of the wheel cycle C_n of wheel molecular graph W_n, The resulting molecular graph is a subdivision molecular graph, called gear wheel molecular graph, denoted as $\tilde{W}_n$.

In this paper, we present the fourth Atom-Bond Connectivity index of $I_r(F_n)$, $I_r(W_n)$, $I_r(\tilde{F}_n)$ and $I_r(\tilde{W}_n)$. Also, the fifth geometric-arithmetic index and its general version of $I_r(F_n)$, $I_r(W_n)$, $I_r(\tilde{F}_n)$ and $I_r(\tilde{W}_n)$ are derived.

II FOURTH ATOM-BOND CONNECTIVITY INDEX

Theorem1.

$$ABC_4(I_r(F_n)) = r\sqrt{\frac{(4n-4)+r(n+2)}{(n+r)((3n-2)+r(n+1))}} + 2\sqrt{\frac{(4n-1)+r(n+4)}{((3n-2)+r(n+1))(n+3r+3)}} +2\sqrt{\frac{(4n+1)+r(n+5)}{((3n-2)+r(n+1))(n+4r+5)}}+(n-4)\sqrt{\frac{(4n+2)+r(n+5)}{((3n-2)+r(n+1))(n+4r+6)}} + 2\sqrt{\frac{2n+8r+9}{(n+4r+5)(n+4r+6)}}+\frac{n-3}{n+4r+6}\sqrt{2n+8r+10} + 2r\sqrt{\frac{n+4r+3}{(n+3r+3)(2+r)}} + 2r\sqrt{\frac{n+5r+6}{(n+4r+5)(3+r)}}+(n-4)r\sqrt{\frac{n+5r+7}{(n+4r+6)(3+r)}}.$$

Proof. Let $P_n=v_1v_2\dots v_n$ and the r hanging vertices of v_i be v_i^1, v_i^2,…, v_i^r $(1\le i\le n)$. Let v be a vertex in F_n beside P_n, and the r hanging vertices of v be v^1, v^2, …, v^r. By the definition of fourth atom-bond connectivity index, we have

$$ABC_4(I_r(F_n)) = \sum_{i=1}^{r}\sqrt{\frac{S(v)+S(v^i)-2}{S(v)S(v^i)}} + \sum_{i=1}^{n}\sqrt{\frac{S(v)+S(v_i)-2}{S(v)S(v_i)}} + \sum_{i=1}^{n-1}\sqrt{\frac{S(v_i)+S(v_{i+1})-2}{S(v_i)S(v_{i+1})}} + \sum_{i=1}^{n}\sum_{j=1}^{r}\sqrt{\frac{S(v_i)+S(v_i^j)-2}{S(v_i)S(v_i^j)}} =$$

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$$r\sqrt{\frac{(4n-4)+r(n+2)}{(n+r)((3n-2)+r(n+1))}}$$ \+

$$(2\sqrt{\frac{(4n-1)+r(n+4)}{((3n-2)+r(n+1))(n+3r+3)}}$$

$$+2\sqrt{\frac{(4n+1)+r(n+5)}{((3n-2)+r(n+1))(n+4r+5)}}$$

$$+(n-4)\sqrt{\frac{(4n+2)+r(n+5)}{((3n-2)+r(n+1))(n+4r+6)}}) +$$

$$(2\sqrt{\frac{2n+8r+9}{(n+4r+5)(n+4r+6)}}+(n-3)\sqrt{\frac{2n+8r+10}{(n+4r+6)(n+4r+6)}})$$

$$+ (2r\sqrt{\frac{n+4r+3}{(n+3r+3)(2+r)}}$$

$$2r\sqrt{\frac{n+5r+6}{(n+4r+5)(3+r)}}+(n-4)r\sqrt{\frac{n+5r+7}{(n+4r+6)(3+r)}})$$

. □

Corollary1.

$$ABC_4(F_n) = 2\sqrt{\frac{4n-1}{(3n-2)(n+3)}}$$

$$+2\sqrt{\frac{4n+1}{(3n-2)(n+5)}}+(n-4)\sqrt{\frac{4n+2}{(3n-2)(n+6)}} +$$

$$2\sqrt{\frac{2n+9}{(n+5)(n+6)}}+\frac{n-3}{n+6}\sqrt{2n+10}.$$

Theorem 2.

$$ABC_4(I_r(W_n)) = r\sqrt{\frac{r(n+2)+4n-2}{(r(n+1)+3n)(n+r)}} +$$

$$n\sqrt{\frac{r(n+5)+4n+4}{(r(n+1)+3n)(n+4r+6)}} +$$

$$\frac{n}{n+4r+6}\sqrt{2n+8r+10}+nr\sqrt{\frac{n+5r+7}{(n+4r+6)(r+3)}}.$$

Proof. Let $C_n=v_1v_2\ldots v_n$ and v_i^1, v_i^2,…, v_i^r be the r hanging vertices of v_i ($1\le i\le n$). Let v be a vertex in W_n beside C_n, and v^1, v^2, …, v^r be the r hanging vertices of v. According to the definition of fourth atom-bond connectivity index, we get

$$ABC_4(I_r(W_n)) = \sum_{i=1}^{r}\sqrt{\frac{S(v)+S(v^i)-2}{S(v)S(v^i)}} +$$

$$\sum_{i=1}^{n}\sqrt{\frac{S(v)+S(v_i)-2}{S(v)S(v_i)}} + \sum_{i=1}^{n}\sqrt{\frac{S(v_i)+S(v_{i+1})-2}{S(v_i)S(v_{i+1})}}$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{r}\sqrt{\frac{S(v_i)+S(v_i^j)-2}{S(v_i)S(v_i^j)}}$$

$$= r\sqrt{\frac{r(n+2)+4n-2}{(r(n+1)+3n)(n+r)}} +$$

$$n\sqrt{\frac{r(n+5)+4n+4}{(r(n+1)+3n)(n+4r+6)}} +$$

$$n\sqrt{\frac{2n+8r+10}{(n+4r+6)(n+4r+6)}}+nr\sqrt{\frac{n+5r+7}{(n+4r+6)(r+3)}}.$$

□

Corollary2.

$$ABC_4(W_n)=n\sqrt{\frac{4n+4}{3n(n+6)}}+\frac{n}{n+6}\sqrt{2n+10}.$$

Theorem3.

$$ABC_4(I_r(\tilde{F}_n)) =$$

$$r\sqrt{\frac{(4n-4)+r(n+2)}{(n+r)((3n-2)+r(n+1))}} +$$

$$2\sqrt{\frac{(4n-2)+r(n+4)}{((3n-2)+r(n+1))(n+3r+2)}}$$

$$+(n-2)\sqrt{\frac{4n+r(n+5)}{((3n-2)+r(n+1))(n+4r+4)}} +$$

$$2r\sqrt{\frac{n+4r+2}{(n+3r+2)(2+r)}}+(n-2)r\sqrt{\frac{n+5r+5}{(n+4r+4)(r+3)}} +$$

$$2\sqrt{\frac{n+6r+5}{(n+3r+2)(3r+5)}}$$

$$+2(n-3)\sqrt{\frac{n+7r+8}{(n+4r+4)(3r+6)}}+2\sqrt{\frac{n+7r+7}{(n+4r+4)(3r+5)}})$$

$$+2r\sqrt{\frac{4r+5}{(3r+5)(r+2)}}+(n-2)r\sqrt{\frac{4r+6}{(3r+6)(r+2)}}.$$

Proof. Let $P_n=v_1v_2\ldots v_n$ and $v_{i,i+1}$ be the adding vertex between v_i and v_{i+1}. Let v_i^1, v_i^2,…, v_i^r be the r hanging vertices of v_i ($1\le i\le n$). Let $v_{i,i+1}^1$, $v_{i,i+1}^2$,…, $v_{i,i+1}^r$ be the r hanging vertices of $v_{i,i+1}$ ($1\le i\le n$-1). Let v be a vertex in F_n beside P_n, and the r hanging vertices of v be v^1, v^2, …, v^r. By virtue of the definition of fourth atom-bond connectivity index, we obtain

$$ABC_4(I_r(\tilde{F}_n)) = \sum_{i=1}^{r}\sqrt{\frac{S(v)+S(v^i)-2}{S(v)S(v^i)}} +$$

$$\sum_{i=1}^{n}\sqrt{\frac{S(v)+S(v_i)-2}{S(v)S(v_i)}}+\sum_{i=1}^{n}\sum_{j=1}^{r}\sqrt{\frac{S(v_i)+S(v_i^j)-2}{S(v_i)S(v_i^j)}}$$

$$+\sum_{i=1}^{n-1}\sqrt{\frac{S(v_i)+S(v_{i,i+1})-2}{S(v_i)S(v_{i,i+1})}}+\sum_{i=1}^{n-1}\sqrt{\frac{S(v_{i,i+1})+S(v_{i+1})-2}{S(v_{i,i+1})S(v_{i+1})}}$$

$$+\sum_{i=1}^{n-1}\sum_{j=1}^{r}\sqrt{\frac{S(v_{i,i+1})+S(v_{i,i+1}^j)-2}{S(v_{i,i+1})S(v_{i,i+1}^j)}}$$

$$= r\sqrt{\frac{(4n-4)+r(n+2)}{(n+r)((3n-2)+r(n+1))}} +$$
$$(2\sqrt{\frac{(4n-2)+r(n+4)}{((3n-2)+r(n+1))(n+3r+2)}}$$
$$+(n-2)\sqrt{\frac{4n+r(n+5)}{((3n-2)+r(n+1))(n+4r+4)}}) +$$
$$(2r\sqrt{\frac{n+4r+2}{(n+3r+2)(2+r)}} +(n-2)r\sqrt{\frac{n+5r+5}{(n+4r+4)(r+3)}}) +$$
$$(\sqrt{\frac{n+6r+5}{(n+3r+2)(3r+5)}}$$
$$+(n-3)\sqrt{\frac{n+7r+8}{(n+4r+4)(3r+6)}}+\sqrt{\frac{n+7r+7}{(n+4r+4)(3r+5)}})$$
$$+(\sqrt{\frac{n+6r+5}{(n+3r+2)(3r+5)}}$$
$$+(n-3)\sqrt{\frac{n+7r+8}{(n+4r+4)(3r+6)}}+\sqrt{\frac{n+7r+7}{(n+4r+4)(3r+5)}}) +$$
$$(2r\sqrt{\frac{4r+5}{(3r+5)(r+2)}}+(n-2)r\sqrt{\frac{4r+6}{(3r+6)(r+2)}}).\ \square$$

Corollary3.

$$ABC_4(\tilde{F}_n) = 2\sqrt{\frac{4n-2}{(3n-2)(n+2)}}$$
$$+(n-2)\sqrt{\frac{4n}{(3n-2)(n+4)}}+2\sqrt{\frac{n+5}{5(n+2)}}$$
$$+2(n-3)\sqrt{\frac{n+8}{6(n+4)}}+2\sqrt{\frac{n+7}{5(n+4)}}).$$

Theorem4.

$$ABC_4(I_r(\tilde{W}_n)) = r\sqrt{\frac{r(n+2)+4n-2}{(r(n+1)+3n)(n+r)}} +$$
$$n\sqrt{\frac{r(n+5)+4n+4}{(r(n+1)+3n)(n+4r+6)}} +$$
$$nr\sqrt{\frac{n+5r+7}{(n+4r+6)(r+3)}} +2n\sqrt{\frac{n+7r+10}{(n+4r+6)(3r+6)}} +$$
$$nr\sqrt{\frac{4r+6}{(3r+6)(r+2)}}.$$

Proof. Let $C_n=v_1v_2\ldots v_n$ and v be a vertex in W_n beside C_n, $v_{i,i+1}$ be the adding vertex between v_i and v_{i+1}. Let v^1, v^2, …, v^r be the r hanging vertices of v and v_i^1, v_i^2,…, v_i^r be the r hanging vertices of v_i $(1\le i\le n)$. Let $v_{n,n+1}=v_{1,n}$ and $v_{i,i+1}^1$, $v_{i,i+1}^2$,…, $v_{i,i+1}^r$ be the r hanging vertices of $v_{i,i+1}$ $(1\le i\le n)$. In view of the definition of fourth atom-bond connectivity index, we deduce

$$ABC_4(I_r(\tilde{W}_n)) = \sum_{i=1}^{r}\sqrt{\frac{S(v)+S(v^i)-2}{S(v)S(v^i)}} +$$
$$\sum_{i=1}^{n}\sqrt{\frac{S(v)+S(v_i)-2}{S(v)S(v_i)}}+\sum_{i=1}^{n}\sum_{j=1}^{r}\sqrt{\frac{S(v_i)+S(v_i^j)-2}{S(v_i)S(v_i^j)}}$$
$$+\sum_{i=1}^{n}\sqrt{\frac{S(v_i)+S(v_{i,i+1})-2}{S(v_i)S(v_{i,i+1})}}+\sum_{i=1}^{n}\sqrt{\frac{S(v_{i,i+1})+S(v_{i+1})-2}{S(v_{i,i+1})S(v_{i+1})}}$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{r}\sqrt{\frac{S(v_{i,i+1})+S(v_{i,i+1}^j)-2}{S(v_{i,i+1})S(v_{i,i+1}^j)}}$$
$$= r\sqrt{\frac{r(n+2)+4n-2}{(r(n+1)+3n)(n+r)}} +$$
$$n\sqrt{\frac{r(n+5)+4n+4}{(r(n+1)+3n)(n+4r+6)}} +$$
$$nr\sqrt{\frac{n+5r+7}{(n+4r+6)(r+3)}} + n\sqrt{\frac{n+7r+10}{(n+4r+6)(3r+6)}}$$
$$+n\sqrt{\frac{n+7r+10}{(n+4r+6)(3r+6)}} + nr\sqrt{\frac{4r+6}{(3r+6)(r+2)}}.\ \square$$

Corollary4. $ABC_4(\tilde{W}_n) = n\sqrt{\frac{4n+4}{3n(n+6)}}$

$$+2n\sqrt{\frac{n+10}{6(n+6)}}.$$

III GENERAL FIFTH GEOMETRIC-ARITHMETIC INDEX

The terminologies for these special molecular graphs similar as Theorem 1- Theorem 4.

Theorem5.

$$GA_5^k(I_r(F_n)) =$$
$$r(\frac{2\sqrt{(n+r)((3n-2)+r(n+1))}}{(4n-2)+r(n+2)})^k +$$
$$2(\frac{2\sqrt{((3n-2)+r(n+1))(n+3r+3)}}{(4n+1)+r(n+4)})^k$$
$$+2(\frac{2\sqrt{((3n-2)+r(n+1))(n+4r+5)}}{(4n+3)+r(n+5)})^k+(n-4)(\frac{2\sqrt{((3n-2)+r(n+1))(n+4r+6)}}{(4n+4)+r(n+5)})^k$$
$$+$$
$$2(\frac{2\sqrt{(n+4r+5)(n+4r+6)}}{2n+8r+11})^k+(n-3)(\frac{2\sqrt{(n+4r+6)(n+4r+6)}}{2n+8r+12})^k$$
$$+ 2r(\frac{2\sqrt{(n+3r+3)(2+r)}}{n+4r+5})^k$$
$$+$$
$$2r(\frac{2\sqrt{(n+4r+5)(3+r)}}{n+5r+8})^k+(n-4)r(\frac{2\sqrt{(n+4r+6)(3+r)}}{n+5r+9})^k$$

.

Proof. By the definition of general fifth geometric-arithmetic index, we have

$$GA_5^k(I_r(F_n)) = \sum_{i=1}^{r}\left(\frac{2\sqrt{S(v)S(v^i)}}{S(v)+S(v^i)}\right)^k + \sum_{i=1}^{n}\left(\frac{2\sqrt{S(v)S(v_i)}}{S(v)+S(v_i)}\right)^k + \sum_{i=1}^{n-1}\left(\frac{2\sqrt{S(v_i)S(v_{i+1})}}{S(v_i)+S(v_{i+1})}\right)^k + \sum_{i=1}^{n}\sum_{j=1}^{r}\left(\frac{2\sqrt{S(v_i)S(v_i^j)}}{S(v_i)+S(v_i^j)}\right)^k$$

$$= r\left(\frac{2\sqrt{(n+r)((3n-2)+r(n+1))}}{(4n-2)+r(n+2)}\right)^k + (2\left(\frac{2\sqrt{((3n-2)+r(n+1))(n+3r+3)}}{(4n+1)+r(n+4)}\right)^k$$
$$+2\left(\frac{2\sqrt{((3n-2)+r(n+1))(n+4r+5)}}{(4n+3)+r(n+5)}\right)^k+(n-4)\left(\frac{2\sqrt{((3n-2)+r(n+1))(n+4r+6)}}{(4n+4)+r(n+5)}\right)^k) +$$
$$(2\left(\frac{2\sqrt{(n+4r+5)(n+4r+6)}}{2n+8r+11}\right)^k+(n-3)\left(\frac{2\sqrt{(n+4r+6)(n+4r+6)}}{2n+8r+12}\right)^k)$$
$$+(2r\left(\frac{2\sqrt{(n+3r+3)(2+r)}}{n+4r+5}\right)^k + 2r\left(\frac{2\sqrt{(n+4r+5)(3+r)}}{n+5r+8}\right)^k+(n-4)r\left(\frac{2\sqrt{(n+4r+6)(3+r)}}{n+5r+9}\right)^k).$$

□

Corollary5. $GA_5^k(F_n) = 2\left(\frac{2\sqrt{(3n-2)(n+3)}}{4n+1}\right)^k +2\left(\frac{2\sqrt{(3n-2)(n+5)}}{4n+3}\right)^k+(n-4)\left(\frac{2\sqrt{(3n-2)(n+6)}}{4n+4}\right)^k + 2\left(\frac{2\sqrt{(n+5)(n+6)}}{2n+11}\right)^k+(n-3)\left(\frac{2\sqrt{(n+6)(n+6)}}{2n+12}\right)^k$.

Theorem6. $GA_5^k(I_r(W_n)) = r\left(\frac{2\sqrt{(r(n+1)+3n)(n+r)}}{r(n+2)+4n}\right)^k + n\left(\frac{2\sqrt{(r(n+1)+3n)(n+4r+6)}}{r(n+5)+4n+6}\right)^k + n + nr\left(\frac{2\sqrt{(n+4r+6)(r+3)}}{n+5r+9}\right)^k$.

Proof. By the definition of general fifth geometric-arithmetic index, we have

$$GA_5^k(I_r(W_n)) = \sum_{i=1}^{r}\left(\frac{2\sqrt{S(v)S(v^i)}}{S(v)+S(v^i)}\right)^k + \sum_{i=1}^{n}\left(\frac{2\sqrt{S(v)S(v_i)}}{S(v)+S(v_i)}\right)^k + \sum_{i=1}^{n}\left(\frac{2\sqrt{S(v_i)S(v_{i+1})}}{S(v_i)+S(v_{i+1})}\right)^k + \sum_{i=1}^{n}\sum_{j=1}^{r}\left(\frac{2\sqrt{S(v_i)S(v_i^j)}}{S(v_i)+S(v_i^j)}\right)^k$$

$$= r\left(\frac{2\sqrt{(r(n+1)+3n)(n+r)}}{r(n+2)+4n}\right)^k + n\left(\frac{2\sqrt{(r(n+1)+3n)(n+4r+6)}}{r(n+5)+4n+6}\right)^k + n\left(\frac{2\sqrt{(n+4r+6)(n+4r+6)}}{2n+8r+12}\right)^k + nr\left(\frac{2\sqrt{(n+4r+6)(r+3)}}{n+5r+9}\right)^k.$$

□

Corollary 6. $GA_5^k(W_n) = n\left(\frac{2\sqrt{3n(n+6)}}{4n+6}\right)^k + n$.

Theorem 7. $GA_5^k(I_r(\tilde{F}_n)) = r\left(\frac{2\sqrt{(n+r)((3n-2)+r(n+1))}}{(4n-2)+r(n+2)}\right)^k + 2\left(\frac{2\sqrt{((3n-2)+r(n+1))(n+3r+2)}}{4n+r(n+4)}\right)^k +(n-2)\left(\frac{2\sqrt{((3n-2)+r(n+1))(n+4r+4)}}{(4n+2)+r(n+5)}\right)^k + 2r\left(\frac{2\sqrt{(n+3r+2)(2+r)}}{n+4r+4}\right)^k +(n-2)r\left(\frac{2\sqrt{(n+4r+4)(r+3)}}{n+5r+7}\right)^k + \left(\frac{2\sqrt{(n+3r+2)(3r+5)}}{n+6r+7}\right)^k +(n-3)\left(\frac{2\sqrt{(n+4r+4)(3r+6)}}{n+7r+10}\right)^k +\left(\frac{2\sqrt{(n+4r+4)(3r+5)}}{n+7r+9}\right)^k + \left(\frac{2\sqrt{(n+3r+2)(3r+5)}}{n+6r+7}\right)^k +(n-3)\left(\frac{2\sqrt{(n+4r+4)(3r+6)}}{n+7r+10}\right)^k +\left(\frac{2\sqrt{(n+4r+4)(3r+5)}}{n+7r+9}\right)^k + 2r\left(\frac{2\sqrt{(3r+5)(r+2)}}{4r+7}\right)^k+(n-2)r\left(\frac{2\sqrt{(3r+6)(r+2)}}{4r+8}\right)^k$.

Proof. By virtue of the definition of general fifth geometric-arithmetic index, we get

$$GA_5^k(I_r(\tilde{F}_n)) = \sum_{i=1}^{r}\left(\frac{2\sqrt{S(v)S(v^i)}}{S(v)+S(v^i)}\right)^k + \sum_{i=1}^{n}\left(\frac{2\sqrt{S(v)S(v_i)}}{S(v)+S(v_i)}\right)^k + \sum_{i=1}^{n}\sum_{j=1}^{r}\left(\frac{2\sqrt{S(v_i)S(v_i^j)}}{S(v_i)+S(v_i^j)}\right)^k +$$

$$\sum_{i=1}^{n-1}\left(\frac{2\sqrt{S(v_i)S(v_{i,i+1})}}{S(v_i)+S(v_{i,i+1})}\right)^k + \sum_{i=1}^{n-1}\left(\frac{2\sqrt{S(v_{i,i+1})S(v_{i+1})}}{S(v_{i,i+1})+S(v_{i+1})}\right)^k + \sum_{i=1}^{n-1}\sum_{j=1}^{r}\left(\frac{2\sqrt{S(v_{i,i+1})S(v^j_{i,i+1})}}{S(v_{i,i+1})+S(v^j_{i,i+1})}\right)^k$$

$$= r\left(\frac{2\sqrt{(n+r)((3n-2)+r(n+1))}}{(4n-2)+r(n+2)}\right)^k + \left(2\left(\frac{2\sqrt{((3n-2)+r(n+1))(n+3r+2)}}{4n+r(n+4)}\right)^k + (n-2)\left(\frac{2\sqrt{((3n-2)+r(n+1))(n+4r+4)}}{(4n+2)+r(n+5)}\right)^k\right)$$
$$+ \left(2r\left(\frac{2\sqrt{(n+3r+2)(2+r)}}{n+4r+4}\right)^k + (n-2)r\left(\frac{2\sqrt{(n+4r+4)(r+3)}}{n+5r+7}\right)^k\right)$$
$$+ \left(\left(\frac{2\sqrt{(n+3r+2)(3r+5)}}{n+6r+7}\right)^k + (n-3)\left(\frac{2\sqrt{(n+4r+4)(3r+6)}}{n+7r+10}\right)^k + \left(\frac{2\sqrt{(n+4r+4)(3r+5)}}{n+7r+9}\right)^k\right)$$
$$+ \left(\left(\frac{2\sqrt{(n+3r+2)(3r+5)}}{n+6r+7}\right)^k + (n-3)\left(\frac{2\sqrt{(n+4r+4)(3r+6)}}{n+7r+10}\right)^k + \left(\frac{2\sqrt{(n+4r+4)(3r+5)}}{n+7r+9}\right)^k\right)$$
$$+ \left(2r\left(\frac{2\sqrt{(3r+5)(r+2)}}{4r+7}\right)^k + (n-2)r\left(\frac{2\sqrt{(3r+6)(r+2)}}{4r+8}\right)^k\right).$$

□

Corollary7.
$$GA_5^k(\tilde{F}_n) = 2\left(\frac{2\sqrt{(3n-2)(n+2)}}{4n}\right)^k + (n-2)\left(\frac{2\sqrt{(3n-2)(n+4)}}{4n+2}\right)^k + \left(\frac{2\sqrt{5(n+2)}}{n+7}\right)^k + 2(n-3)\left(\frac{2\sqrt{6(n+4)}}{n+10}\right)^k + 2\left(\frac{2\sqrt{5(n+4)}}{n+9}\right)^k + \left(\frac{2\sqrt{5(n+2)}}{n+7}\right)^k.$$

Theorem8.
$$GA_5^k(I_r(\tilde{W}_n)) = r\left(\frac{2\sqrt{(r(n+1)+3n)(n+r)}}{r(n+2)+4n}\right)^k + n\left(\frac{2\sqrt{(r(n+1)+3n)(n+4r+6)}}{r(n+5)+4n+6}\right)^k + nr\left(\frac{2\sqrt{(n+4r+6)(r+3)}}{n+5r+9}\right)^k + 2n\left(\frac{2\sqrt{(n+4r+6)(3r+6)}}{n+7r+12}\right)^k + nr\left(\frac{2\sqrt{(3r+6)(r+2)}}{4r+8}\right)^k.$$

Proof. In view of the definition of general fifth geometric-arithmetic index, we deduce

$$GA_5^k(I_r(\tilde{W}_n)) = \sum_{i=1}^{r}\left(\frac{2\sqrt{S(v)S(v^i)}}{S(v)+S(v^i)}\right)^k + \sum_{i=1}^{n}\left(\frac{2\sqrt{S(v)S(v_i)}}{S(v)+S(v_i)}\right)^k + \sum_{i=1}^{n}\sum_{j=1}^{r}\left(\frac{2\sqrt{S(v_i)S(v_i^j)}}{S(v_i)+S(v_i^j)}\right)^k + \sum_{i=1}^{n}\left(\frac{2\sqrt{S(v_i)S(v_{i,i+1})}}{S(v_i)+S(v_{i,i+1})}\right)^k + \sum_{i=1}^{n}\left(\frac{2\sqrt{S(v_{i,i+1})S(v_{i+1})}}{S(v_{i,i+1})+S(v_{i+1})}\right)^k + \sum_{i=1}^{n}\sum_{j=1}^{r}\left(\frac{2\sqrt{S(v_{i,i+1})S(v^j_{i,i+1})}}{S(v_{i,i+1})+S(v^j_{i,i+1})}\right)^k$$

$$= r\left(\frac{2\sqrt{(r(n+1)+3n)(n+r)}}{r(n+2)+4n}\right)^k + n\left(\frac{2\sqrt{(r(n+1)+3n)(n+4r+6)}}{r(n+5)+4n+6}\right)^k + nr\left(\frac{2\sqrt{(n+4r+6)(r+3)}}{n+5r+9}\right)^k + n\left(\frac{2\sqrt{(n+4r+6)(3r+6)}}{n+7r+12}\right)^k + n\left(\frac{2\sqrt{(n+4r+6)(3r+6)}}{n+7r+12}\right)^k + nr\left(\frac{2\sqrt{(3r+6)(r+2)}}{4r+8}\right)^k.$$ □

Corollary8.
$$GA_5^k(\tilde{W}_n) = n\left(\frac{2\sqrt{3n(n+6)}}{4n+6}\right)^k + 2n\left(\frac{2\sqrt{6(n+6)}}{n+12}\right)^k.$$

IV FIFTH GEOMETRIC-ARITHMETIC INDEX

By taking k=1 in Theorem 5-8, we get the following conclusions on fifth geometric-arithmetic index.

Theorem9.

$$GA_5(I_r(F_n)) = \frac{2r\sqrt{(n+r)((3n-2)+r(n+1))}}{(4n-2)+r(n+2)} + \frac{4\sqrt{((3n-2)+r(n+1))(n+3r+3)}}{(4n+1)+r(n+4)} + \frac{4\sqrt{((3n-2)+r(n+1))(n+4r+5)}}{(4n+3)+r(n+5)} + \frac{2(n-4)\sqrt{((3n-2)+r(n+1))(n+4r+6)}}{(4n+4)+r(n+5)} + \frac{4\sqrt{(n+4r+5)(n+4r+6)}}{2n+8r+11} + \frac{2(n-3)\sqrt{(n+4r+6)(n+4r+6)}}{2n+8r+12} +$$

$$\frac{4r\sqrt{(n+3r+3)(2+r)}}{n+4r+5}+\frac{4r\sqrt{(n+4r+5)(3+r)}}{n+5r+8}+\frac{2(n-4)r\sqrt{(n+4r+6)(3+r)}}{n+5r+9}.$$

Corollary9.

$$GA_5(F_n)=\frac{4\sqrt{(3n-2)(n+3)}}{4n+1}+\frac{4\sqrt{(3n-2)(n+5)}}{4n+3}+\frac{2(n-4)\sqrt{(3n-2)(n+6)}}{4n+4}+\frac{4\sqrt{(n+5)(n+6)}}{2n+11}+\frac{2(n-3)(n+6)}{2n+12}.$$

Theorem 10.

$$GA_5(I_r(W_n))=\frac{2r\sqrt{(r(n+1)+3n)(n+r)}}{r(n+2)+4n}+\frac{2n\sqrt{(r(n+1)+3n)(n+4r+6)}}{r(n+5)+4n+6}+n+\frac{2nr\sqrt{(n+4r+6)(r+3)}}{n+5r+9}.$$

Corollary 10. $GA_5(W_n)=\frac{2n\sqrt{3n(n+6)}}{4n+6}+n$.

Theorem 11.

$$GA_5(I_r(\tilde{F}_n))=\frac{2r\sqrt{(n+r)((3n-2)+r(n+1))}}{(4n-2)+r(n+2)}+\frac{4\sqrt{((3n-2)+r(n+1))(n+3r+2)}}{4n+r(n+4)}+\frac{2(n-2)\sqrt{((3n-2)+r(n+1))(n+4r+4)}}{(4n+2)+r(n+5)}+\frac{4r\sqrt{(n+3r+2)(2+r)}}{n+4r+4}+\frac{2(n-2)r\sqrt{(n+4r+4)(r+3)}}{n+5r+7}+\frac{2\sqrt{(n+3r+2)(3r+5)}}{n+6r+7}+\frac{2(n-3)\sqrt{(n+4r+4)(3r+6)}}{n+7r+10}+\frac{2\sqrt{(n+4r+4)(3r+5)}}{n+7r+9}+\frac{2\sqrt{(n+3r+2)(3r+5)}}{n+6r+7}+\frac{2(n-3)\sqrt{(n+4r+4)(3r+6)}}{n+7r+10}+\frac{2\sqrt{(n+4r+4)(3r+5)}}{n+7r+9}$$

$$+\frac{4r\sqrt{(3r+5)(r+2)}}{4r+7}+\frac{2(n-2)r\sqrt{(3r+6)(r+2)}}{4r+8}.$$

Corollary11. $GA_5(\tilde{F}_n)=$

$$\frac{\sqrt{(3n-2)(n+2)}}{n}+\frac{(n-2)\sqrt{(3n-2)(n+4)}}{2n+1}+\frac{2\sqrt{5(n+2)}}{n+7}+\frac{2(n-3)\sqrt{6(n+4)}}{n+10}+\frac{2\sqrt{5(n+4)}}{n+9}+\frac{2\sqrt{5(n+2)}}{n+7}+\frac{2(n-3)\sqrt{6(n+4)}}{n+10}+\frac{2\sqrt{5(n+4)}}{n+9}.$$

Theorem12.

$$GA_5(I_r(\tilde{W}_n))=\frac{2r\sqrt{(r(n+1)+3n)(n+r)}}{r(n+2)+4n}+\frac{2n\sqrt{(r(n+1)+3n)(n+4r+6)}}{r(n+5)+4n+6}+\frac{2nr\sqrt{(n+4r+6)(r+3)}}{n+5r+9}+\frac{4n\sqrt{(n+4r+6)(3r+6)}}{n+7r+12}+\frac{2nr\sqrt{(3r+6)(r+2)}}{4r+8}.$$

Corollary 12. $GA_5(\tilde{W}_n)=\frac{2n\sqrt{3n(n+6)}}{4n+6}+\frac{2n\sqrt{6(n+6)}}{n+12}+\frac{2n\sqrt{6(n+6)}}{n+12}$.

V. CONCLUSION

In this paper, we determine the fourth atom bond connectivity index and fifth geometric-arithmetic index of fan molecular graph, wheel molecular graph, gear fan molecular graph, gear wheel molecular graph, and their r-corona molecular graphs. Furthermore, the general version of fifth geometric-arithmetic index is discussed.

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